Assignment 13.

1. Solve the quadratic equation $z^2 + (1+\mathrm{i})z + (-6-2\mathrm{i}) = 0$. [5]

2. The complex number z is given by $z = -1 + \sqrt{3}\mathrm{i}$. Find $|z|$ and $\arg z$. [2]

   Hence express $w = \dfrac{z}{2z^*}$ in the form of $a + b\mathrm{i}$, where a and b are real. [3]

3. Sketch, on a single Argand diagram, the loci of points representing the complex number z defined by each of the following conditions: [1][1][2]

   (i) $|z - 2\mathrm{i}| = 2$, (ii) $\arg(z+2) = \frac{1}{4}\pi$, (iii) $|z-2| \leq |z+2\mathrm{i}|$.

   Find the complex number z that satisfies all the conditions of (i), (ii) and (iii), expressing your answer in the form of $a + b\mathrm{i}$, where a and b are real and in exact forms. [3]

4. The variable complex number z is given by $z = \sin\theta + \mathrm{i}(1 - \cos\theta)$, where θ takes all values in the interval $[0, 2\pi)$.

(a) Show that $|z - \mathrm{i}| = 1$, for all values of θ. [2]

(b) Hence sketch, in an Argand diagram, the locus of the point representing z. [2]

(c) Show also that $0 \leq |z| \leq 2$, for all values of θ. [2]

(d) Express $\arg z$ in terms of θ, simplifying your answer as much as possible. [4]

(e) Find the least value of $\arg(z + 3 + \mathrm{i})$, correct to 3 decimal places. [3]

5. (†) Let ω be a non-real fifth root of unity: $\omega^5 = 1$, and $\omega \notin \mathbb{R}$.

(a) Show that $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$. [2]

(b) Use the result in part (a) to show that $\cos\left(\frac{2}{5}\pi\right) + \cos\left(\frac{4}{5}\pi\right) = -\frac{1}{2}$. [3]

(c) Hence find the exact value of $\cos\left(\frac{2}{5}\pi\right)$. [3]

Total mark of this assignment: $30 + 8$.

The symbol (†) indicates a bonus question. Finish other questions before working on this one.